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ABSTRACT

Assuming guided inquiry as a pedagogical ideal in mathematics education implies that teaching must connect students' thinking about a subject with curricular agendas and instructional goals. Because students are not typically inclined to consider their active inquiry as a route to acquiring the knowledge that is valued in school, such teaching must simultaneously elicit students' engagement in inquiry and legitimate inquiry as a route to learning. Within this conceptual framework, the empirical research reported in this paper describes several strategies used by secondary school geometry teachers as they attempted to practice a pedagogy of guided inquiry using the "Geometric Supposers." The teachers' strategies are discussed in terms of sociolinguistic theories about the teacher's role in defining the meaning of mathematical knowledge in the classroom. (Author)

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Technical Report

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TECHNICAL REPORT

October 1988

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ABSTRACT

Assuming guided inquiry as a pedagogical ideal in mathematics education implies that teaching must connect students' thinking about a subject with curricular agendas and instructional goals. Because students are not typically inclined to consider their active inquiry as a route to acquiring the knowledge that is valued in school, such teaching must simultaneously elicit students' engagement in inquiry and legitimate inquiry as a route to learning. Within this conceptual framework, the empirical research reported in this paper describes several strategies used by secondary school geometry teachers as they attempted to practice a pedagogy of guided inquiry using the *Geometric Supposers*. The teachers' strategies are discussed in terms of sociolinguistic theories about the teacher's role in defining the meaning of mathematical knowledge in the classroom.

Over the past five years, the Educational Technology Center at Harvard Graduate School of Education has been developing tools for teachers and students to use in the service of supporting "guided inquiry" in school. Guided inquiry is an approach to curriculum and instruction which gives the teacher the responsibility for introducing content in a way that is illuminated and modified in response to students' ways of thinking about that content. The teacher defines the focus of inquiry by posing problems for the class while students take an active part in acquiring knowledge by generating not only answers, but ways of thinking about problems, definitions of the terms of discourse, and analyses of alternative solutions. Students collect data and analyze it, and what the teacher aims to teach is supplemented and complemented by the discussion of student findings. This is the pedagogical ideal that has guided the Technology Center's work in research and development. It is also an ideal embraced by mathematics educators more generally in the U.S. and abroad.

One of the tools developed at ETC to help teachers turn this ideal of guided inquiry into a classroom reality is a set of problems designed to accompany *The Geometric Supposers*, computer software which can be used to construct and measure geometric figures, namely triangles, quadrilaterals, and circles (Yerushalmy and Houde, 1986; Schwartz, Yerushalmy, and Gordon, 1985). The problems are intended for use by students, working alone or in pairs at computer terminals, and they are also adaptable to class-sized discussions with the teacher working at a large monitor at the front of the room. By providing teachers and students with the facility to quickly produce, measure, and compare figures, the software supports the process of making conjectures about spatial relationships based on induction as a prior step toward establishing the generality of the relationships with a deductive proof. Beginning with induction contributes to an environment in which students may be more likely to bring their own observations to bear on the construction of mathematical relationships, and to see "proof" as an appropriate step for making a particular observation into a general assertion. The *Supposers* are used in many secondary school classrooms across the country, and ETC has been working closely with teachers in some of those classrooms to better understand the problems that teachers face as they try to implement curriculum and instruction based on the principle of guided inquiry (Wiske, 1988; Wiske and Houde, 1988).

Although many teachers in schools may try to accommodate students by responding to their questions at the end of a lesson or even organizing classes around small group discussions, the ideal of guided inquiry tips the balance even further toward treating students as active shapers of their own knowledge, while teachers work to direct their activity toward commonly recognized curricular goals. The pedagogy of guided inquiry has been studied at ETC with a focus on using the

Supposers in high school geometry, but what is being learned in this project is not only relevant to the study of teaching that subject or to understanding the implementation of a new technology. The work of supporting student's inquiry in school classrooms has been undertaken at many levels of mathematics instruction, and in other subject matter areas as well. And yet the teaching strategies that make up this kind of practice are not very well understood.

One of the most persistent questions that arises when teachers try to take account of students thinking is how to organize lessons in a way that connects the questions that students care to pursue with the goals of teachers and the schools in which they work (Petrie, 1981; Berlak & Berlak. 1981; Lemke, 1982; Barnes, 1976). The issue of connecting students' active inquiry with teachers' agendas is a broad one, spanning not only the mathematics curricula in primary and secondary school, but other school subjects as well (Romberg & Carpenter, 1986; Lampert, 1988b; Brophy, in press; Lochhead & Clement, 1979; Sinclair, 1988; Cobb, 1988; Schoenfeld, in press). Whether one begins with a curriculum and asks how it is possible to get students interested in asking productive questions, or one begins with students' questions and asks how teachers and schools can go about shaping them into a coherent program of curriculum and instruction, there is a connection problem. From the teacher's point of view, solving this problem depends on the development of teaching strategies that are effective in maintaining both students' active engagement with subject matter and the accomplishment of institutional goals. Such strategies need not be invented by teachers, but they need to be such that teachers are both capable and disposed to employ them. The research to be reported in this paper is an exploration of the strategies teachers using the *Geometric Supposers* have tried to use to solve the "connection problem".

THE NATURE OF THE "CONNECTION" PROBLEM

Making students' inquiry a legitimate way of knowing

The problem of inventing teaching strategies that will connect student thinking with teachers' agendas is not only a problem of how to teach subject matter in a way that takes account of what students bring. It is also a problem of communicating with students in a way that legitimates their active involvement in creating their own knowledge (Cazden, 1988). In addition to whatever understanding of the mathematics content students bring to learning mathematics, they also bring assumptions about what they are supposed to do and what teachers are supposed to do to cause their learning to happen (Ball, 1988; Cooney, 1987).

One of the teachers who had been using the *Supposers* in her Geometry class for a few months surveyed her students to find out what they thought about using the computer software to produce and collect data from which they could make conjectures about geometrical relationships. The students had been spending one or two days a week in the school's computer lab, exploring definitions and problems with the constructions they produced on the computer screen. When they were back

in class, on the other days of the week, the teacher attempted to integrate the findings of their explorations with discussions, lectures, and homework assignments that were organized according to a standard textbook. For some of the students, this integration was experienced in the way it was intended, as an attempt to engage them in actively constructing their own knowledge. One young woman member of the class wrote:

> Instead of just feeding us the information and expecting us to understand it instantly, [the Supposer] makes us draw our own figures in our own ways and makes us figure out the questions by ourselves. It makes us use our brains a little more; for example, to draw a figure, we must use some property of it which we then learn much better, since we must use it for our construction. As we are doing our own little things in creative ways it is forcing us to discover new geometry facts and conjectures. We are, in a way, with the teacher's guidance, teaching ourselves... It is also sort of rewarding when you've discovered something, and you, for a short fleeting moment, feel like a genius.

This student's sense of what was happening to her in her high school geometry class parallels what epistemologists call coming to "know in the strong sense" (Scheffler, 1965). As she directed the computer to make the constructions posed as problems, she found out about properties of figures and which properties are consistent across different versions of a figure. The student was not only acquiring information, she was acquiring the conviction that what she was learning is true by verifying it for herself, and she was sorting out important information from the superficial characteristics of figures. She was, with the teacher's guidance, "teaching herself" geometry, and learning that mathematical knowledge is knowledge that can be discovered inductively.

But this student's sense of how one learns mathematics is unusual, both in classes using the *Supposers*, and in mathematics classes more generally. More typical is the student who writes:

> I feel that the *Supposer* could be of help if it was used in conjunction with a teacher who also re-taught the material, making sure everyone knew exactly what was going on. What would be good would be to have the teacher teach the material, then have students see it for themselves on the computer. But if the teacher teaches the material, then why bother with the *Supposer*?... A good teacher emphasizes important facts; minor things that you would spend four or five periods on in the computer room can be emphasized in one class, and move on to more important information. I feel that time spent in the computer room has been pretty much a waste.

This student believed it was the teacher's job to figure out what he needed to know, and that he would learn most efficiently if the teacher told him what was important to know. He did not want to "waste time" exploring problems in order to figure out for himself what facts were important. He saw Geometry as a subject which can be

transmitted from teacher to student in a direct form, rather than as a subject in which the important facts are determined by the perspective of the learner, working on the sorts of problems the subject was invented to solve. Although this student's beliefs would not be considered productive by either educational philosophers or cognitive psychologists, his sense of what knowledge is and how it is acquired fits very well into the way teaching and learning are usually organized in school, and it also fits with commonly held assumptions about what kind of knowledge mathematics represents.

Non-mathematical ways of knowing school mathematics

Commonly, mathematics is associated with remembering what to do; knowing it, with being able to get the right answer, quickly . These cultural assumptions are reinforc‹ l by school experience, in which doing mathematics means following the rules laid down by the teacher; knowing mathematics means remembering and applying the correct rule when the teacher asks a question; and mathematical truth is determined when the answer is ratified by the teacher (Stodolsky, 1988). These beliefs about how to do mathematics and what it means to know it in school are acquired through years of watching, listening, and practicing (Carpenter, Corbitt, Kepner, Lindquist, & Reys, 1981; Dossey, Mullis, Linquist, and Chambers, 1988). It is probably the prevalence of such beliefs that prompts the young woman quoted above to experience the exaggerated feeling that she must be a "genius" if she can actually figure something out for herself in a mathematics class. Schoenfeld (1985a) argues that most students approach the study of mathematics with a theory of knowledge that works against their taking an active role in figuring out why what they are learning is true. This theory of knowledge or "epistemology" is non-mathematical in the sense that it contradicts the very foundation on which mathematical knowledge is built. Coming to know that a mathematical assertion is true *within mathematics* is a matter of deriving conclusions from assumptions by a process of plausible reasoning (Polya, 1954; Lakatos, 1976; Kramer, 1970). But instead of seeing mathematics as a discipline whose assertions are the result of reasoned argument, and therefore discoverable, students think of mathematical invention as the province of a few "geniuses"-- and they rarely count themselves as members of that group. They believe that:

> Only geniuses are capable of discovering mathematics [so] If you forget something, too bad. After all, you're not a genius and you won't be able to derive it on your own... Accept procedures at face value, and don't try to understand why they work. After all, they are derived knowledge passed on 'from above'. (Schoenfeld, 1985 a, p. 372)

That many students hold this theory of knowledge and act on it as they are learning mathematics is confirmed by several empirical studies (Schoenfeld, 1985b; Stodolsky, 1985; Ball, 1988). It is not inconsistent that the same students learn facts and rules well enough to pass the sorts of tests their teachers administer, even though they are

missing what is commonly referred to as "understanding". They do not have the knowledge to explain why mathematical facts are true or why the rules they apply to solve problems work (Schoenfeld, 1987).

How is mathematical knowledge acquired?

Psychologists and philosophers would take exception with the idea that only "geniuses" are capable of discovering and understanding why the mathematical facts and procedures they learn in school make sense. Scholars in both of these traditions assert that the quality and availability of knowledge depends on the active part that the learner plays in acquiring it, figuring out for him or her self what is true (e.g. Van Lehn, 1986; Resnick, in press; Scheffler, 1965; Peters, 1967). Knowledge which is "fed" to the learner is hardly considered to be knowledge at all: "knowing in the strong sense is more than just true belief, involving also the ability to justify or back up the belief in an appropriate manner." (Scheffler, 1965, p. 55) Learning depends on learners of a discipline knowing strategies which they can use to evaluate whether or not an assertion is true; it is more than memorizing what is passed down "from above". According to cognitive theory, learning occurs when the learner considers new knowledge in light of what he or she already knows to be true, and constructs a relationship between the new knowledge and the old (Anderson, 1981; Van Lehn, 1986; Hiebert, 1987; Schoenfeld, 1987b). It is not necessary for students to discover the whole of mathematics in order to acquire new knowledge, but they do need to take a critical attitude toward what they are told is true by teachers and books, reconstructing information to make sense of it on their terms. The teacher's role is to provide the environment in which the student can "confront reality for himself" (Scheffler, ibid.).

And yet the common assumptions that underlie how people think about mathematical knowledge and its acquisition depend heavily on the authority of teachers and books to define mathematical "reality". These authorities are considered by learners to be reliable sources of information about what is worth knowing and legitimate standards against which to measure whether they are learning what is important. As a respected "record of knowledge", the contents of a textbook are taken as a given. memorized, and repeated back on tests (Romberg, 1983). For the most part, neither classroom interactions nor written assessments of students' knowledge are designed to find out whether students have subjected the facts and rules they are taught to the sort of critical evaluation that psychologists and philosophers consider prerequisite to learning. Some scholars have asserted that it is the very fact that such evaluation is missing in schools that makes it unlikely that students will undertake serious inquiry (Bereiter and Scardamalia, 1986; Cuban, 1985).

The problem of teaching students to be active learners

This is the "problem space" within which teachers who want to enact a pedagogy of guided inquiry do their work. If their students are going to learn what they need to know through engaging in guided inquiry, teachers must teach in a way that manages the contradiction between these common assumptions about intellectual authority and the idea that mathematical knowledge can and should be actively constructed as it is learned. And they must do this even as the traditional norms of the situation in which they teach work against students taking their own thinking seriously as a route to being successful (Cuban, 1985; Cohen, 1988). Recent comprehensive studies of high schools suggest that few high school students are sitting around just dying to figure out what they think about relationships among geometric figures, and even the few who do are not likely to come into a Geometry class assuming that what they think has much to do with what is important for them to learn (Powell, Farrar, & Cohen, 1985). The findings of the developers of the *Geometric Supposer* materials concur with these general studies; for example, they concluded from an extensive set of interviews with students who had used the materials for a year that

> ...students prefer to learn in ways that require less work. Thus some students prefer a traditional classroom where the teacher does a few examples of a kind of problem and then homework is to do a set of problems like those done in class. (Yerushalmy, Chazen, & Gordon, 1988, p. 23)

Other studies of particular educational innovations that are built on the assumption that the ideal student is curious and actively engaged in inquiry come to similar conclusions about students' propensities to become active learners in school (Doyle, 1986; Stephens, 1982; Brophy, in press).

So then what is the nature of the problem that needs to be addressed by teachers who wish to enact a pedagogy of guided inquiry? If a teacher chooses to challenge students' expectations about the kinds of academic tasks that constitute their educational program -- and I have argued here that this would be a necessary part of engaging students in this sort of learning process -- then teaching strategies need to be employed which not only CONNECT student inquiry with teachers' curricular agendas, but also ELICIT acts of inquiry on which to build these connections, and LEGITIMATE inquiry as a process that will result in students learning what they need to know to be successful in school. The strategies that will be described here were used by teachers as they tried to accomplish these three goals simultaneously.

How do teachers organize and teach lessons so that learners can and will explore a topic from the perspective of their own understanding and teachers can and will take account of that understanding in what and how they teach? How can intellectual and social coherence be maintained in classrooms where teachers do let students' ways of thinking about mathematics become a substantial part of the agenda of mathematics lessons? How can a different social structure be created in the

classroom, so that the teacher retains social and intellectual authority, but also takes seriously the need for students to verify knowledge for themselves in order to learn what is being taught? How can teachers attend to the predictably diverse ways of understanding a mathematical idea that students will bring to a topic, and also conduct lessons that are orderly and predictable enough to fit the school's institutional structure? How can students' inquiry be plausibly linked with the content goals that are assigned to schools? These are large questions which have been on the minds of educational reformers for more than a hundred years. The research to be reported here makes a modest stab at addressing them by looking at strategies teachers have used to address the three-part goal of eliciting and legitimating students' inquiry in the field of geometry, and connecting that inquiry with curricular goals.

DATA COLLECTION AND ANALYSIS

An advisor, Richard Houde, was employed part-time by ETC at the beginning of the 1986-87 school year to consult with a "Users Group" of six teachers who were experimenting with the *Geometric Supposers* in three different high schools (small rural, large urban, and affluent suburban) to teach geometry with a pedagogy of guided inquiry. [See Shepard & Wiske (in press) for a fuller description of the teachers and schools.] He observed and advised these teachers on a regular basis during two school years. Some of the consultations were short term visits, wherein he would spend a half day in a school every few weeks. Others were more extensive, consisting of daily visits to one teacher over several days, during which they would plan, carry out, and evaluate an activity together. Houde was also a Geometry teacher and Mathematics Department Head, and he retained this position half time, simultaneously with his advisory work.

During the 1987-88 school year, Houde also took on the role of an "action researcher" (Lewin, 1948), in addition to being an advisor. As a participant observer in these six teachers' classrooms during that year, he collected extensive fieldnotes, which included descriptions of the strategies the teachers attempted in their effort to build lessons in response to students' thinking, as well as records of his own interventions. His perspective was that of a teacher and a teacher supervisor, and thus he brings a practical rather than a theoretical bias to research. In addition, classroom observations of the same teachers were conducted by the author over three week-long periods during 1986-87 and 1987-88. Observational field notes were supplemented by audiotapes collected by the author in two teachers' classrooms. Field notes were expanded immediately after each class session observed, using the audiotapes when available, to ensure as complete and accurate a record as possible of the teachers' actions in the classroom. Pre-observation and post-observation interviews were conducted with each of the teachers by the author. These interviews were structured to address directly the teachers plans to employ strategies that were intended to connect student thinking with the curriculum, as well as the teachers' post-lesson reflections on those strategies. All interviews were tape-recorded and later transcribed.

The question that guided this data collection effort was: What practices intended to support guided inquiry are *possible* under the constraints of ordinary classroom teaching? In addition, the data analysis identified teaching strategies that were designed to communicate to students that their own inquiry was considered to be a legitimate source of knowledge about geometry in school. Observation and interview data collected by Houde and the author were analyzed to identify strategies which the teachers used. No attempt was made to evaluate whether the strategies attempted were effective. [1] Strategies which were tried once were included in the analysis as well as those which occurred on a regular basis

These methods were designed to capture *the practitioner's perpective* on a teaching innovation that has been favored by educational theory and research. Data collected by the teacher-advisor was filtered through his role as a consultant who was regularly invited by teachers to help them figure out how to do what they wanted to do. The teacher participants believed that guided inquiry was an appropriate approach to teaching and learning geometry, and that the *Supposers* were an appropriate tool to support that approach (Lampert, 1988a). But they had many questions about how to act on their beliefs in the context of their responsibilities for curriculum and instruction. The strategies that they used to make students' thinking a more central part of lesson agendas were often collaboratively developed with the advisor and undertaken with his encouragement. Teachers and advisor jointly evaluated the teacher's practices and devised revisions in strategies. The reliability of the advisor's reports was checked by the author's observations and interviews with the teachers.

The intent was to capture practices that were experimental attempts to teach in a way that tips the balance of classroom activity more heavily in the direction of student's taking an active role in producing the knowledge they acquire; this approach might be called "transformative" research from the perspective of current reform agendas (Silver, in press). It was assumed that these practices would be a practical adaptation of theoretical ideals, compromises worked out by teachers to fit the circumstances of their work (Wiske & Houde, 1988; Lampert, 1985; Crosswhite, 1987). In practice, teachers must often compromise to manage all aspects of their work (Brophy, 1988), and we know little about the extent to which ideals -- like guided inquiry -- can be retained in those compromises. The *Supposers* constituted a technological intervention which invited students to be more active in acquiring knowledge about geometry, and gave them a means for becoming so; at the same time, the innovation gave teachers new problems with which to cope. More of the students' naive knowledge of geometry was exposed, and the teachers were confronted with figuring out ways to build on this knowledge while at the same time being responsible for the curriculum.

[1] Such research awaits the design of adequate tools for measuring the extent to which students' own knowledge results from constructing a connection between their own active inquiry and the content of the curriculum that the teacher is teaching.

The teachers who experimented with the *Supposers*, like many others, began from the pedagogical baseline of lecturing and answering students questions as if the knowledge of the subject matter could be transferred directly from books or syllabi to students trough lectures and practice with the problems at the end of each chapter in the textbook. Whatever compromises they made to accommodate guided inquiry thus represents a considerable shift in practice toward the ideal. In the literature on mathematics education, we have many treatises on what is wrong with current teaching methods (e.g. Whitney, 1986; Steen, 1988), and a few portraits of what ideal teaching might look like (e.g. Fawcett, 1934). But these portraits are suspect because they are not derived from teaching that occurs in ordinary circumstances (Crosswhite, 1987). The strategies that are described here are meant to be something of an "existence proof" to contradict the notion that it is not possible for ordinary teachers to incorporate students' thinking into classroom practice, but they also illustrate the practical compromises that are made when ideals are transformed into reality.

TEACHING STRATEGIES

The "Three Blackboard" Method

Perhaps the most straightforward attempt to make a relationship between students' individual thinking and the formal curriculum in teaching practice was the "three blackboard" method which some *Supposer* teachers attempted to use. This teaching routine made a visible connection between the results of the relatively private inquiry that occurred when students worked in pairs at the computer and the public discourse of the class lecture-discussion. Used primarily to follow a computer lab session, this strategy had the teacher writing on the board, first with the intention of simply taking down what students said in answer to a general question about the lab activity like, "What did you notice?" or "What patterns did you see in the data?" After several students' findings were written down in the student's words for every one else to see, the teacher asked students to make these observations into more formal assertions about the figures they had been constructing and measuring. These assertions were written down on the second blackboard. Often a student formalized his or her own observations, but once informal statements were written on the blackboard, they were considered to be available for others in the class to think about and formalize, as well. Some teachers wrote students' names next to both informal and formal assertions, and referred to a conjecture verbally as belonging to the student who offered it in class, saying for example, "Tomorrow we're going to work on *Sandy's* conjecture about the ratio of the sides." or "I want you to try and prove *Denise's* conjecture about parallelism for homework." Some of the time it was the teacher who translated informal assertions into potentially provable conjectures. In the transition from first to second blackboard, the teacher may also have intervened with the introduction of new conventions of terminology or labeling, or reminded students to reword their thinking in terms of mathematical conventions with which they were already familiar.

In the transition from the second to the third blackboard, the teachers took a more directive role, choosing one of the formalized student conjectures and writing

it as it might be found in a geometry textbook. At this point, conjectures were formulated in ways that would lead to the deductive arguments that turn them into theorems and they were made to look like the standard statements found in books. In one teacher's classroom, what was written up on the third blackboard, and then proven, in lecture, discussion, or assignment form, was then considered to be a statement that could be used as a reason in a subsequent proof. Students and teacher would refer to such assertions in terms of whether they have been "done on the blackboard yet" using this contextual distinction to stand in for the formal distinction between a conjecture based on induction and a theorem whose truth has been established with a publicly constructed deductive proof.

From pairs to paper

Another strategy for surfacing and using students' thinking was to have them work in pairs in the lab to produce a single report of their observations. The information recorded on the "third blackboard" was actually several steps away from the student's inquiry as it occurred in the lab, and at each step, the language and symbols for expressing what the student might "know" about a figure became more and more conventional, until they resembled the language and symbols used in the textbook. The most undirected student work that occurred in most of the *Supposer* user's classrooms ~~occurred~~ in the computer lab, where students usually worked at terminals in pairs.

The teaching strategy of having students work in pairs sets up a learning situation in which individuals would make assertions and other individuals would challenge them because the two students needed to agree about what would be written down on paper. The first step from individual thinking to mathematical discourse was taken when the pairs of students *talked* with one another about what they thought might be patterns in the measurements they had taken. Then as they worked together on deciding "what to put down" on paper while pointing to figures on the screen, they negotiated with one another about what might be worth saying about the figures, developing their own idiosyncratic language for communicating about geometric properties and figures.

Because of time constraints, students' written lab reports were usually finished as a homework assignment and then their papers were brought to class for discussion. This step meant that individuals would review what they had discussed with their partners in the lab, and refine it further. Putting findings into a form that would be "handed in" meant that they were challenged to communicate their observations in terms that would be meaningful to the teacher. Some of the teachers had students read from their papers in the class discussion of the lab work, but others had students turn in their papers, putting off the discussion for a few days until they read them over to plan a class lecture or discussion. Although some of the teachers thought this latter approach would better enable them to distill students' findings into a coherent lesson, they also thought that something of what they called the "students' ownership of the ideas" was lost if too many days passed between informal

informal inquiry in pairs and its formalization as part of the public agenda for the whole class. Often this time lag was forced by the logistics of scheduling lab time and the unpredictability of the school schedule.

Transporting students ideas from one class to another

One routine that the *Supposer* teachers devised to cope with this time lag was to use conjectures that students came up with in one class group as the basis for a discussion in another class group. Particularly in the second year of using the *Supposers*, the teachers had some idea of the kinds of conjectures that students were likely to come up with, given a particular lab problem. So if they intended to teach a lesson that followed from discoveries that students might make in the lab, but they could not schedule the exploration to coincide with their lesson plan, they would use one class's lab reports as the basis for moving another class from discoveries to theorems. For example, one of the teachers began a class by saying, "A student in my fourth period came up with this method for constructing a similar triangle inside of another triangle, so that the corresponding sides are parallel. Why do you think it works?"

Here student inquiry as a route to acquiring important knowledge was legitimized, even though the teacher was not using the thinking of a member of the class she was teaching as the basis for discussion. Especially when students were experienced users of the *Supposers*, they responded to this strategy as if they could readily imagine themselves coming up with an assertion like the one the teacher reported as having come from another class. They seemed to be challenged by the idea that someone in another class came up with an idea that the teacher was impressed with, and there was some competitive spirit involved in moving it to the next step by producing a proof of why a particular construction "worked". Competition entered in another way, as students speculated about the teacher possibly taking *their* conjectures into another class to be proven.

Teacher interaction with conjecturing pairs

When students were working in the computer lab, they were spread around the periphery of the room facing computer screens, with their backs to the center of the room. This made it difficult for the teacher to address the class as a whole, or to use the blackboard in a didactic way. When the teachers wanted to give or clarify a direction, some of them would walk around the room instead of talking to students' backs or causing physical disruption by having them turn their chairs around, saying the same thing repeatedly to different pairs of students. When students had something to say to the teacher, this format made their assertion less of a public performance and more like sharing an idea with a collaborator.

Because the teacher represents intellectual authority in school, the classroom agenda is formally set by what she is saying to whom. If this talk occurs to pairs of

students, and it has academic substance, then what the teacher can communicate is that what the pairs are doing is as important to their learning as what happens when everyone in the class is supposed to be listening to the teacher. Although it was exhausting and time consuming, this strategy enabled the teacher to time the giving of directions differently for different students, and it had the effect of legitimizing the private, inquiry driven activity that was going on between pairs of students.

Since the *Supposer* produces different figures for each pair of students when they direct it to construct, say, an acute triangle, the teacher talking with pairs is even more particularized. What she says to one pair to get them to focus their observations and conjectures on a specific part of the figure will be different from what she says to others, and this interaction has the further potential of communicating to students that it is appropriate that not everyone is doing the same thing at the same time to learn what it is important to know to be a successful geometry student.

Teacher and students challenging one another to "Prove it"

The way that teachers and students interacted around subject matter was quite dramatically affected by the fact that students could get access to many examples of a geometric relationship that were not given to them by a teacher or book. Teachers and books might instruct by stating an abstract principle, like a definition or a theorem, first and then they may or may not give a particular example to help "explain" the principle. In the *Supposer* labs, students could generate example after example without any such abstraction to organize their perceptions. To the extent that they were looking for something as they generated these examples, the something they were looking for was an abstraction of their own construction.

When a pair of students found a mathematical relationship among figures that seemed generalizable, they were directed to make another construction on the screen with similar characteristics and find out if the relationship would hold for that construction. This is the formal procedure to follow if one is being strictly inductive about generating an assertion. But in actuality, students were often so certain that the relationship they had found in one example was universal that they said they did not need to try it out again to know that it was always true. This would evoke the challenge, "Prove it!" either from the teacher or from peers, and thus students would be drawn into the deductive process by the kinds of interactions that were engendered by *Supposer* problems.

Having established this way of interacting informally in the lab, teachers were able to use the "Prove it!" refrain when students made a similar generalization in a class discussion. In lab sessions, the idea of logical argument in support of assertion had been separated somewhat from the formality of the proofs that appeared in textbooks in two columns of "statements" and "reasons", and this set the stage for teachers being able to ask students to generate informal explanations of why an assertion made sense *to them* in their own terms, without being overwhelmed by the

standard for deductive argument set by the textbook.[2] In most classrooms, this sort of discussion occurred and incorporated student inquiry into the agenda, but it often was conducted along side of rather than integrated with teaching and learning about formal proof.

Fudging on grades

In order to alleviate their students' worries about evaluation enough to get them to be willing to risk making conjectures, the teachers made various kinds of accommodations in their grading systems as a way of teaching students something about the difference between inquiry and authority as sources of knowledge. All of the teachers had given homework, quizzes and tests in the past which assessed students' abilities to produce conventionally correct definitions and conventionally structured proofs for theorems that used correct statements and reasons. But they could not evaluate conjectures generated from a lab activity on the same sort of "correct vs. incorrect" standard. Not having enough evidence does not make a conjecture "wrong" in the same way that not having appropriate reasons makes a proof "wrong". Throughout the two years that the *Supposer* experiment occurred in these classrooms, teachers and students struggled with how to evaluate students' conjectures – but that is another story (Cf. Wiske and Houde, 1988).

In terms of teaching strategies used to elicit and connect student thinking with the curricular agenda, what can be said here is that the teachers relaxed their grading standards in an attempt to give students room to experiment with ideas that might or might not turn out to lead in fruitful directions. They tried to be flexible in the ways in which they took account of both the quantity and the quality of the thinking students did about what they observed in the computer lab. Students did not have the choice of *not* writing observations or conjectures, but a wide range of productions were taken as acceptable.

CASES OF STRATEGY USE

In order to put these disembodied strategies in some context, three lessons will be described here that illustrate the sort of teaching that happened in the *Supposer* classrooms. As these cases show, the separate strategies described above are woven together in practice and adjusted to the subject matter and the overall style and circumstances of the teacher who is using them. The lessons described here are

[2] The leap from induction to deduction in these kinds of exchanges was a messy one, more like that which characterizes work in mathematics itself than that which characterizes mathematical pedagogy (Davis and Hersh, 1987), and similar to that which is found when students are given the task of proving that a mathematical assertion is true to their peers outside of the classroom context (Balachef, 1988).

typical of lessons taught by each of the teachers. Classroom lessons, rather than lab sessions, are used here as illustrations because they have more potential for having been taught under circumstances one would consider "ordinary" in secondary schools.

Making definitions

At the beginning of the school year, one of the teachers planned a lab session and subsequent class discussions with two major goals in mind, one a more conventional curriculum related goal and the other oriented to students learning something about their own role in generating mathematical knowledge.[3] He wanted his students to learn how to distinguish different kinds of triangles from one another, and he wanted them to learn the conventional terms that are used to refer to the different kinds of triangles. But he also wanted his students to learn that definitions in geometry are invented, and that there was a relationship between the definitions one accepted and the inferences that could be drawn.

The assignment for the lab session was very open-ended. Students were told to "Find out everything you can about different kinds of triangles by measuring angles and sides, and make a list of the fact that you think are true about different kinds of triangles."[4] In the class after this lab, the teacher announced that he was going to "list all your facts about triangles on the board" and then the class would "discuss what makes a good definition." As he began making the list, it was difficult

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[3] This is what ETC calls a "metacurricular goal" (Position Paper, 1987) and others call "metacognition" or "epistemic" learning (e.g. Schoenfeld, 1987b).

[4] The *Supposer* for triangles has a menu that allows the student to command the computer to draw six different kinds of triangles: right, acute, obtuse, isosceles, equilateral, and "your own." If the student selects the "your own" option, then the message at the bottom of the screen will change to:

1 side	2 side	3 angle
side	angle	side
side	side	angle

$0 < side < 9$

allowing three different ways to specify the kind of triangle to be drawn.

to hold students back from commenting on one another's "facts". The class became a quick and lively exchange of assertions, counterassertions, and evidence, during which the teacher consciously collected information about "what these students already knew" about the properties of geometrical figures. Once there was a long list of findings up on the board, written in language close to that used by the students who asserted them, the teacher observed, speaking to the students, that some of their "facts" focused on the relationships among *angles* in a triangle, and some focused on relationships among *sides*.

For homework, after this class discussion, the teacher assigned his students to pick either "angles" or "sides" and rewrite all of their "facts" using only one or the other characteristic. He used this work as the basis for a lecture in the next class, where he demonstrated that what was true about the sides could be implied from what was true about the angles, and vice versa. He then challenged the class to think about "what makes a good definition." During the discussion, students began to speculate about combinations of characteristics, asking questions like, "Can you have a right triangle that is also isosceles?" or "Does an equilateral triangle have to be acute?" The students did not arrive at any consensus about what a good definition should include, and the way that the discussion ended represented something about the difficulties of teaching in a way that respects the thinking that students bring to understanding a subject. One of the students asserted, "We would not be having all this trouble if we just used the book definitions." Several other members of the class agreed, and said that book definitions were better than any they could come up with on their own. Since this discussion pressed well beyond the "bell" that signaled the end of Geometry period, the teacher closed the discussion with the promise that they would return to this issue several more times in the course of the year.

Conjecturing in class

In a school where few classes include any sort of discussion, one of the *Supposer* teachers led a post-lab session in which students were expected to find patterns in the data they had collected about medians and altitudes in different kinds of triangles. She began the class by asking three students to go to the board, draw their lab drawings (sketches of what the *Supposer* had produced on the screen), and list their data. Where data was incomplete, other students were asked to fill in. She did not diverge from the lesson at that point to talk about standards of completion, but kept the class focused on looking for patterns in the data by asking them what observations they could make about "relationships" between parts of figures. She restated her question as "What did you get for conjectures?", indirectly teaching students that their observations about relationships are what are called conjectures, but again, she did not diverge from the content to speak didactically about the nature of conjecturing.

One student offered, "The height is always greatest from vertex to A." The teacher asked the rest of the class, "Look at your papers; did you get that result?" and a discussion ensued in which many informal ways of referring to figures, some

general and other particular were used. The teacher did not infuse the discussion with lessons that would direct students toward referring to "A" in more generic terms. Instead, she kept coming back to the relationship between statements about relationships and evidence in the data. Other assertions were offered by students: "The altitudes in acute triangles are all inside." "In obtuse triangles, one altitude is inside, and the other two are outside." To such assertions, the teacher responded with questions like, "What is your evidence?" "Did anyone else find a similar relationship?" "Does that always work?" and then, "*Why do you think that will always be true?*" In this way, the class discussion moved from assertions based on induction to more general statements based on arguments about plausible generalizations.

When students began to lose interest in the discussion, the teacher turned on the large monitor in the front of the room and booted the *Supposer* disc for quadrilaterals, challenging the students to extend their conjectures about triangles to four-sided figures. At this point in the lesson, she also tried to draw in students who had not been active participants in the earlier discussion, one of whom said straightforwardly, "I'm not good at this conjecturing stuff." She modeled the process for the more reticent students so that they could make some assertions without producing the whole conjecture themselves.

"Proving it" as a part of ordinary classroom discourse

With a group of students who were further along in their study of Geometry, a teacher wrote on the board, "If ratios of corresponding sides are congruent, then the triangles are similar." Without saying anything about proof, she then asked a student to "Give me a small triangle." The student responded "4,5,2" following a routine that had obviously been established in previous interactions. The teacher then went to the *Supposer* on the computer in the front of the room and constructed a triangle with sides of the lengths she had been given by the student. (The *Supposer* constructs on the basis of a unit that is approximately one-tenth of the width of the screen in length.) She then went to the board and sketched the "4,5,2" triangle to look like the one on the *Supposer* screen, and labeled the lengths of the sides as 4, 5, and 2. Next to it she drew a larger similar-looking triangle, and labeled its corresponding sides 8, 10, and 4. She asked the class, "What do we have to know if these triangles are similar?" A student responded, "Congruent angles." In the context of the class, this shorthand was a way of establishing that in order *to prove* that the two triangles were similar, it would need to be established the the corresponding angles of the two triangles were congruent.

The teacher went to the *Supposer* and used the "Measure" option to determine the measures of each of the angles of the "4,5,2" triangle that had been constructed on the screen. She then constructed another triangle on the screen by typing in the conditions that the sides should measure 8, 10, and 4 units, and measured the angles of this triangle. Many students called out that these angle measures were the same as those for the "4,5,2" triangle. In the midst of this melee,

one student called out above the others, "*Prove it!*" The teacher responded by saying, "Find the proof in your book and read it for homework." She had succeeded in setting the class up for this assignment, and although her response to the student's recognizing the need for proof could have been more creative, she did recognize that the reading of the proof in the book would be enhanced by this dramatic demonstration of how one characteristic of similar triangles implied another.

Going on with the lesson, the teacher asked the students if they thought that having only *two* pairs of corresponding sides proportional was enough to guarantee similar triangles. A student asserted that two pairs of sides was not enough and suggested dimensions for two triangles which the teacher then sketched on the board. One was a "3,4,5" right triangle, and the other was a "6,120 degree, 8" triangle. She asked the other students what they thought, and several concurred that the conjecture was false. Next the teacher posed another conjecture: "What if the angles contained in the two pairs of corresponding sides are equal?" Students drew "test" triangles at their seats while she constructed some using the *Supposer*.

After some discussion and another reference to a proof to read for homework, a student made a conjecture: "Wouldn't the ratios of the areas be the same as the ratios of the sides?" The teacher responded, "Let's see." and went to the *Supposer* again to produce a test case. She constructed two triangles whose corresponding sides were in the ratio 1.5:1, and asked the class, "How many people think the area ratio will be 1.5?" before she calculated it on the computer. One student asserted that the ratio of areas would be "1.5 squared" and another student quickly followed with, "Yeah. That has to be true because the altitudes of the triangles are in the ratio 1.5." The teacher checked out this assertion using the *Supposer* to measure. Then she calculated the ratio between the areas which came out to be 2.25:1.

Moving to a different level of argument, a student then asked, "But have we *proved* that these two triangles are similar?" He was going back to the conjecture about two pairs of corresponding sides with the same ratio and included angles that are equal, and wondering if that *always* would produce similar triangles. The teacher asked the class, "Have we proved it?" There was a chorus of "No" and one student called out: "We haven't done the steps on the board," referring to the teacher's routine for writing out formal arguments, using suggestions from the class, on the board.

DISCUSSION

Teachers' terms of discourse

In the context of the high school Geometry classes that used the Supposers, students' perspectives on spatial relationships were referred to as "observations" or "conjectures" by teachers, students, advisor, and researchers. The distinction between observations and conjectures had to do with the extent to which the student's assertion was tied to particular cases, drawn on the computer screen with one of the Supposers. Observations were particular, and led to inductive statements

about figures. Conjectures were stated in terms of a generalization, and in most classes the next step was to go on to prove that the conjecture was true using deduction on conjectures that had already been proven and agreed upon definitions. These categories were not hard and fast, however, and the terms were often misused by students and sometimes by teachers.

In all of the classes, teacher and students moved, physically and conceptually, between "the lab" and "the classroom". In the lab, students worked in pairs on Supposer problems, in which they were assigned to make various constructions and measure parts of them. They collected data on charts and then looked for patterns. Regularly, the teachers stated that the purpose of this activity was "to come up with your own conjectures" but in actuality, because of time constraints, the work of conjecturing was often given for homework or undertaken outside of the lab, in the classroom. When students and teachers were together in the classroom, their work was guided by a textbook. Sometimes, there was no mention of the Supposer or lab problems, even though teacher and students moved back and forth between these two environments about once a week. More typically, lab problems were discussed in the classroom, and on a few occasions, teachers used the software on a large monitor in front of the room to demonstrate or explore constructions.

The physical division within the school setting between the lab and the classroom was undoubtedly symbolic of the conceptual division between student inquiry and the teacher's agenda. Certainly geographical separation between computer-based activities and regular classroom lessons contributed to the integration problem, but was not entirely responsible for it. On some occasions, teachers undertook lab-like work in the context of a whole class discussion using a large monitor in the classroom, but most admitted to using it more when the advisor was observing than they otherwise would. Easy access to hardware and fluid facility with it might help teachers connect student inquiry with their planned curricula, but discontinuity had other roots as well.

The phrases "connecting the labs with the classroom" and "connecting the labs with the textbook" came up over and over again as the teachers talked about the problems they were having eliciting student's thinking, making inquiry a legitimate route to acquiring knowledge, and connecting student's assertions with the content they wanted them to learn. Referring to the distinct physical spaces became a shorthand for the conceptual dichotomy they were trying to manage. In contrast, the teachers spoke of "conjectures" in a way that blurred the distinction between students' discoveries and the theorems in the textbook. They saw the lab activities as a route to student's acquiring what they called "ownership" of the ideas that they wanted them to learn. For the most part, they designed and directed the labs so that the progression of students' findings would follow the same linear path through Euclidean geometry that was taken in the textbook. [5]

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[5] In notable exception, they sometimes would diverge from the order in the textbook in their second year of using the *Supposer*, because they had observed students taking

Teaching about knowing by structuring interactions

One way to look at the strategies that the *Supposer* teachers used to try to connect student inquiry with curricular agendas and instructional goals is in terms of the ways in which they changed the interaction patterns between teacher and students. Teaching is not only about teaching what is conventionally called "content". It is also teaching students what a lesson is and how to participate in it (Jackson, 1968; Florio, 1978; Mehan, 1979). From the activities the teacher sets for them, students learn what counts as knowledge and what kind of activities constitute legitimate academic tasks (Lemke, 1982; Doyle, 1985, 1986; Leinhardt & Putnam, 1988; Cazden, 1988). Face to face interaction between students and their teacher follows context specific rules, and cues within these contexts signal how what anyone says is to be understood in relation to the task everyone is assembled to accomplish (Mehan, 1979; Cazden, 1988). The teacher has more power over how acts and utterances get interpreted, being in a position of social and intellectual authority, but these interpretations are finally the result of negotiation with students about how activity is to be regarded.

Sociolinguistic research suggests that alterations in patterns of interaction can be initiated by the teacher to build a "participation structure" that redefines the roles and responsibilities of both teacher and students in relation to learning and knowing (Au & Jordan, 1980; Au & Mason, 1981). The notion of a classroom participation structure is taken from the work of Florio (1978) and Erickson and Shultz (1977). They define a participation structure to be the allocation of interactional rights and obligations among participants in a social event; it represents the consensual expectations of the participants about what they are supposed to be doing together, their relative rights and duties in accomplishing tasks, and the range of behaviors appropriate within the event. Teachers and students thus form communities of discourse who come to agree on working definitions of what counts as knowledge and the processes whereby knowledge is assumed to be acquired (Cazden, 1988).

In the classroom, words like "know", "think", "revise", "explain", "problem", and "answer" come to have meaning by being associated with particular kinds of activities. Who is responsible for doing the activities associated with these words gets determined in interaction between the teacher and the students. In conventional lessons the participants agree that it is the teacher's responsibility to *explain* and the students' responsibility to give *answers*. *Problems* are questions, and finding the correct answers to those questions is an indication of *knowing mathematics*. The *teacher* is responsible for knowing whether the answers a student gives are correct and to be asked by the teacher to *revise* means "you've done it

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a different direction in their lab work the year before (Wiske & Houde, 1988). This sort of restructuring of the architecture of the course from one year to the next could be interpreted as a teaching strategy that connects curriculum and student thinking at a larger level than that reported in this paper.

wrong" and "you've got to do it over" until more of the answers are correct. When the teacher asks a students to *think* it is often an admonition to be quiet; thinking is not considered to be a process that students-- or teacher and students-- engage in together.

In order to challenge conventional assumptions about what it means to know mathematics and enact a pedagogy of guided inquiry, teachers and students working with the *Supposers* needed to do different sorts of activities together, with different kinds of roles and responsibilities. As they used the strategies described above, the *Supposer* teachers connected novel, student-initiated activities with words like "know", "think", "revise", "explain", "problem", and "answer". By basing some of their lessons on conjectures derived from students' inductive thinking about the figures they constructed and measured in the computer lab, the teachers legitimated that approach to acquiring and verifying knowledge. They built the formal language of geometry in steps, from the very idiosyncratic and context-embedded pointing that students did in front of the computer screen, to "natural language" descriptions of patterns in the data collected from measuring the figures, to mathematical words, and finally to symbolic expressions. At each step, the transition to a less personal more conventional way of speaking about observations was taken for the purpose of communicating assertions to a wider and wider audience. Thus the students' evolution and expression of mathematical knowledge and conviction followed a path "from pointing to proving" that is similar to the way knowledge grows in the discipline (Freudenthal, 1978, p.242).

Shifting authority by making room for exploratory speech

Another framework for interpreting the *Supposer* teachers' pedagogical strategies, also derived from sociolinguistics, is the distinction between the kinds of thinking that can be ascertained in speech patterns as they occur with different audiences (Barnes, 1976). Simply put, this theory suggests that we speak to an "intimate" audience in an improvised and exploratory way, while speech to a "distant" audience expresses pre-planned and explicitly ordered patterns of thinking. Applied to the *Supposer* classes, the lab activities would fall into the category of intimate communication, and the classroom discussions into the category of speaking to a distant audience. Many of the strategies used by the teachers were intended to break down this distinction, so that students would share their conjectures and argue about them in class discussions, in front of the teacher, as well as with their partners in the lab, and thereby make their thinking a part of the formal agenda.

Barnes observed several instances of small group and whole class interactions in school settings and found:

> many of the children were rearranging their thoughts during improvised talk [in small group discussions]. This did not make for explicit communication, but it played an important part in problem solving... the tentativeness of exploratory talk may for many children

> may be a necessary condition for achieving hypothesis forming and testing.
>
> When teachers entered the groups, asking questions intended to further their pupils' understanding, the style of speech shifted away from exploratory towards a style appropriate to showing the teacher that they had 'the right answer'. The use of exploratory language did not seem to reflect different abilities of particular children but rather the degree of *control over knowledge* which they felt themselves to have. They ceased to use language to shape knowledge for themselves as soon as the authority of the teacher was present. (1976, p. 108)

Although the teachers that Barnes observed did not attempt to change students perceptions of who has control over knowledge, he cites other studies to suggest that making such a change is possible. He asserts that the inexplicitness, confusion, and dead-ends that are a part of exploratory talk, and a condition of student's sense of control over the revised, "final draft" expressions that result from its clarification *can be* a part of teacher-student as well as student-student interaction. There was certainly a tone of exploration such as Barnes describes in the lab sessions as students worked on *Supposer* problems with their peers. The teachers sometimes suceeded, as can be seen from the above cases, in bringing this tone into whole class discussions, and carrying it over to material that was unrelated to *Supposer* problems.

Barnes found that in classrooms where teachers make a distinction between "presenting" talk and "sharing" talk, students were less likely to assume that the teacher was always going to judge what they had to say to be correct or incorrect. In trying to get students to make and critique conjectures in the mathematics classroom, this distinction would be essential. Within mathematics, a *good* conjecture is not necessarily the same thing as a *correct* conjecture. The *Supposer* teachers found it difficult to free students from the worry about whether their conjectures were correct. Like the students that Barnes observed, their students wanted only to say things in whole class discussions that they knew would get the teacher's approval. In order to do this, students formalizes what they "know" to fit what are perceived to be the teachers' standards. But if the teacher responds to student talk in a way that is accepting rather than evaluative, *the student* can retain control over the formalization of his or her own knowledge. Barnes concludes:

> The distinction between exploratory and final draft is essentially a distinction between different ways in which speech can function in the rehearsing of knowledge. In exploratory talk and writing, the learner himself takes responsibility for the adequacy of his thinking; final draft talk and writing looks toward external criteria and distant, unknown audiences. Both uses of language have their place in education... I am emphasizing exploratory language because the social order established in many schools excludes it in favor of final drafts. (1976, p. 114)

What counts as knowledge in school has conventionally only been associated with "final draft" talk, from the teacher's as well as the learner's point of view. Changing that view of knowledge means making room for exploratory talk that is treated seriously and sanctioned as an appropriate way to learn what one needs to know to be successful in school. The two students quoted above (pages 6 and 7) suggest that the *Supposer* teachers have been differentially successful in getting their students to use their own capacity for making sense of geometry as the standard for judging the validity of their knowledge. The student who considers the *Supposer* work to be a waste of time is still relying heavily on the teacher to determine what he needs to know and how he should know it.

The *Supposer* teachers' strategies gave students many opportunities for exploratory talk in the activity of learning geometry. When students talked together at the computer, they were most able to work according to their own, rather than the teacher's standards for appropriateness. When the teacher talked with individual pairs in the labs, students were able to express their conjectures more tentatively than they were able to do in whole class discussions, where they would be publicly judged by both teacher and peers. But even in those whole class discussions, using teacher-to-student and student-to-teacher interaction patterns like challenging one another to "Prove it!" brought more exploratory talk into the situation. Both teacher and students asked "what if" questions, and the thinking that went into rejecting a conjecture was valued as much as the proof of a valid conjecture.

DIRECTIONS FOR FURTHER RESEARCH

The hypothesis that the strategies described above change the learners' sense of his or her role in the learning process could be tested by a careful observational study, taking account of how often the strategies are used and in what configuration, and comparing student interview data before and after being taught with these strategies with data collected in classrooms where such strategies are rarely if ever used. The teachers who have been experimenting with the *Supposer* and developing the strategies along with the project advisor have probably reached a point where they have practiced the strategies enough to participate in such a study. But the simple counting of teacher actions would be unlikely to capture the more subtle shifts in epistemology that have been occurring along with the change in behavior. It would be important in such a research project to take account of teachers' and participants' beliefs about the nature of mathematical knowledge, as well as the strategies they use to enact these beliefs in their practice. The simple introduction of new behaviors, without a complimentary shift in attitudes about what causes and constitutes knowing would be unlikely to have long term effects on either teachers or students (Cf. Stephens, 1982; Cooney, 1987).

On a more pessimistic note, it would be well to examine the practices that *Supposer* teachers and other school teachers use that might counteract the possibility of students taking more responsibility for their own learning. (Assessment practices and tracking, which are partly done by teachers and partly done by the school as an

institution would be possible candidates. See Silver, in press and Cuban, 1985.) These counterproductive practices might be so strong that no matter how genuine the attempts at getting students to take an alternative approach to knowing, they will not counteract the prevailing sense that teachers and textbooks are the ultimate standards against which the validity of students' knowledge is to be judged. The *Supposer* is a powerful tool for teachers to use in the enactment of a pedagogy of guided inquiry, but it may be that there are elements of teacher behavior or school structure that make it so difficult for teachers to use this tool, that it would be unreasonable to expect it to have a major impact.

The National Council of Teachers of Mathematics Commission on Standards for School Mathematics (NCTM, 1987) has asserted that "to gain mathematical power, students need to make conjectures, abstract properties and relationships from problem situations, explain their reasoning, validate assertions, and communicate results in a meaningful form" (p.7). In describing the goals for middle and secondary school students, in particular, the Commission asserted: "Problem solving should be a process that actively engages students in making conjectures, investigating and exploring ideas, discussion and questioning their own thinking and the thinking of othe validating results, and making convincing arguments (p. 54)." Whether thes oals can be achieved in a public education system, and what it takes to achieve them is largely unknown. We need ways of thinking about what students with "mathematical power" would be able to do so that we can know whether attempts achieve such goals are successful. Mathematical power certainly has something to do with the generative nature of mathematical thinking, such as making conjectures and arguing about the extent of their application Lakatos, 1976; Polya, 1954). But very little research has been done which gives attention to what such activities should look like when they are done competently by students in school, or on what they might contribute to the learning of the mathematics we expect students to know at the nd of their coursewo (Silver, in press). The *Supposer* research is a hopeful beginning, establishing that such generative thinking can occur in geometry classrooms in different kinds of ordinary public schools as well as in more precious "demonstration" settings (Cf. Fawcett, 1934; Crosswhite, 1987). We need a better picture of what the "power" might be that students gain from this experience.

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